Example 11: Is the matrix $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -2 & 1 \end{bmatrix}$ invertible? Explain.

$$\det(A) = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 3 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & -1 \\ 2 & 0 & 1 \\ 5 & 0 & -1 \end{vmatrix}$$

$$= (1)(-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 5 & -1 \end{vmatrix} = (-1)(-2-5) = 7.$$

Since $\det(A) \neq 0$, A is invertible by theorem 4.6

Example 12: Is the matrix $A = \begin{bmatrix} 3 & 0 & 3 & -1 \\ 1 & 0 & 2 & 2 \\ 0 & -1 & 1 & 4 \\ 2 & 0 & 1 & -3 \end{bmatrix}$ invertible? Explain.

$$\det(A) = \begin{vmatrix} 3 & 0 & 3 & -1 \\ 1 & 0 & 2 & 2 \\ 0 & -1 & 1 & 4 \\ 2 & 0 & 1 & -3 \end{vmatrix} = (-1)(-1)^{3+2} \begin{vmatrix} 3 & 3 & -1 \\ 1 & 2 & 2 \\ 2 & 1 & -3 \end{vmatrix}$$

$$= (1) \begin{vmatrix} -3 & 0 & 8 \\ -3 & 0 & 8 \\ 2 & 1 & -3 \end{vmatrix} = (1)(-1)^{3+2} \begin{vmatrix} -3 & 8 \\ -3 & 8 \end{vmatrix}$$

$$= (-1)(-24 + 24) = 0$$

$\det(A) = 0$

Since $\det(A) = 0$, the matrix A is not invertible by theorem 4.6